Aufgabe 1

$$\begin{aligned} |45°\rangle\langle 45°| &= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \\ |L\rangle\langle L| &= \frac{1}{2}\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \\ |R\rangle\langle R| &= \frac{1}{2}\begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix} \end{aligned}$$

- R, L, 45:

$$A_{ges} = |45°\rangle\langle 45°|L\rangle \underbrace{\langle L|R\rangle}_{=0}\langle R| = 0$$

$$I_{out} = 0$$

- L, R, 45:

$$A_{ges} = |45°\rangle\langle 45°|R\rangle \underbrace{\langle R|L\rangle}_{=0}\langle L| = 0$$

$$I_{out} = 0$$

- 45, R, L:

$$A_{ges} = |L\rangle \underbrace{\langle L|R\rangle}_{=0}\langle R|45°\rangle\langle 45°| = 0$$

$$I_{out} = 0$$

- R, 45, L:

$$A_{ges} = |L\rangle\langle L|45°\rangle\langle 45°|R\rangle\langle R| = |L\rangle\langle L|\frac{1}{4}\begin{pmatrix} 1+i & 1-i \\ 1+i & 1-i \end{pmatrix} = \frac{1}{4}\begin{pmatrix} i & 1 \\ 1 & -i \end{pmatrix}$$

$$\big||\psi_{out}\rangle\big| = \left|\frac{1}{4}\begin{pmatrix} i & 1 \\ 1 & -i \end{pmatrix}\cdot|\psi_{in}\rangle\right| = \left|\frac{1}{4}\sqrt{2\psi_{in,x}^2 + 2\psi_{in,y}^2}\right| = \frac{1}{\sqrt{8}}\,|\psi_{in}| \quad \Rightarrow \quad I_{out} = \frac{1}{8}I_{in}$$

- 45, L, R:

$$A_{ges} = |R\rangle \underbrace{\langle R|L\rangle}_{=0}\langle L|45°\rangle\langle 45°| = 0$$

$$I_{out} = 0$$

- L, 45, R:

$$|R\rangle\langle R|45°\rangle\langle 45°|L\rangle\langle L| = |R\rangle\langle R|\frac{1}{4}\begin{pmatrix} 1-i & 1+i \\ 1-i & 1+i \end{pmatrix} = \frac{1}{4}\begin{pmatrix} -i & 1 \\ 1 & i \end{pmatrix}$$

$$\big||\psi_{out}\rangle\big| = \left|\frac{1}{4}\begin{pmatrix} -i & 1 \\ 1 & i \end{pmatrix}\cdot|\psi_{in}\rangle\right| = \left|\frac{1}{4}\sqrt{2\psi_{in,x}^2 + 2\psi_{in,y}^2}\right| = \frac{1}{\sqrt{8}}\,|\psi_{in}| \quad \Rightarrow \quad I_{out} = \frac{1}{8}I_{in}$$

Aufgabe 2

a) Dichtematrix S:

$$\begin{aligned} S &= \frac{1}{4}\left[\ |x\rangle\langle x| + |y\rangle\langle y| + |R\rangle\langle R| + |L\rangle\langle L|\ \right] \\ &= \frac{1}{4}\left[\begin{pmatrix}1\\0\end{pmatrix}\cdot\begin{pmatrix}1 & 0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}\cdot\begin{pmatrix}0 & 1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\i\end{pmatrix}\cdot\begin{pmatrix}1 & -i\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\-i\end{pmatrix}\cdot\begin{pmatrix}1 & i\end{pmatrix}\right] \\ &= \frac{1}{4}\left[\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} + \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1 & -i\\i & 1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1 & i\\-i & 1\end{pmatrix}\right] \\ &= \frac{1}{2}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \end{aligned}$$

b) Der Drehimpuls ist 0, da sich die Wellenanteile R und L, die einen Drehimpuls transportieren, gerade aufheben.

Aufgabe 3

$$\begin{aligned} |R\rangle\langle R| + |L\rangle\langle L| &= \frac{1}{2}\begin{pmatrix}1\\i\end{pmatrix}\cdot\begin{pmatrix}1 & -i\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\-i\end{pmatrix}\cdot\begin{pmatrix}1 & i\end{pmatrix} \\ &= \frac{1}{2}\begin{pmatrix}1 & -i\\i & 1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1 & i\\-i & 1\end{pmatrix} \\ &= \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = \mathbb{1} \end{aligned}$$

$$\begin{aligned} \left[\ |R\rangle\langle L| + |L\rangle\langle R|\ \right]^2 &= \left[\frac{1}{2}\begin{pmatrix}1\\i\end{pmatrix}\cdot\begin{pmatrix}1 & i\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\-i\end{pmatrix}\cdot\begin{pmatrix}1 & -i\end{pmatrix}\right]^2 \\ &= \left[\frac{1}{2}\begin{pmatrix}1 & i\\i & -1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1 & -i\\-i & -1\end{pmatrix}\right]^2 \\ &= \left[\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\right]^2 \\ &= \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = \mathbb{1} \end{aligned}$$